# Structural health monitoring in strategies for life-cycle management of dikes: a case study in the north of the Netherlands

W.J. Klerk & W. Kanning *Deltares, Delft, Netherlands* 

M.T. van der Meer Fugro GeoServices B.V., Nieuwegein, Netherlands

J.W. Nieuwenhuis Waterschap Noorderzijlvest, Netherlands

ABSTRACT: In recent years there has been an increasing focus on life-cycle and asset management as a means of managing systems of ageing infrastructure. To accurately assess performance, structural health monitoring can be an important means. In this paper a tool is presented which assesses the costs and benefits of structural health monitoring in life-cycle management strategies for dikes. The tool is based on and applied to the Ommelanderzeedijk, an old sea dike in the north of the Netherlands that didn't pass its last safety assessment. Waterschap Noorderzijlvest has applied several monitoring techniques over the past years, such as piezometric head monitoring and temperature measurements as well as a failure test. These experiments have provided a variety of valuable information, which has led to a more effective design for the upcoming reinforcement. This case focuses on assessing the benefits of monitoring of the phreatic level in the dike body. With the developed tool, various strategies for monitoring can be compared based on their Net Present Value. Although in the actual case the benefits of monitoring were enormous, it is also shows that this can vary greatly depending on the case and that the amount of risk one needs to take in order to obtain information is pivotal in this analysis. However, it can be concluded that structural health monitoring should be an important part of life-cycle management of flood defences.

# **1 INTRODUCTION**

# 1.1 Life-cycle management of dikes

Life-cycle management concerns the management of an asset during its entire life-cycle, and is an important field of interest within the broader field of asset management. In life-cycle management, an asset is considered from 'cradle-to-grave', meaning that all phases of the life-cycle are considered integrally (Labuschagne and Brent, 2005). For dikes, this poses a bit of a problem, as dikes generally do not have a grave but are reinforced, however, if the life-cycle is formulated such that the 'grave' is the reached once the dike is disapproved, and the 'cradle' starts with designing and planning the reinforcement, life-cycle management is applicable to dikes. This is the definition used in this paper. When considering the performance of a dike over its lifecycle, this improves the solution space for maintaining performance, as it integrally considers reinforcement, maintenance and also structural health monitoring (SHM) as a means of reducing uncertainty, which improves the efficiency of measures. In

this paper different strategies for reinforcement and structural health monitoring are considered in one framework, and assessed based on their performance, risk and cost over a period of multiple lifecycles.

# 1.2 The case of the Ommelanderzeedijk

In 2011, after the Ommelanderzeedijk was assessed unsafe in the latest safety assessment, it was decided to make it one of the first LiveDikes in the Netherlands. This means that from that time various types of (live) structural health monitoring were applied and tested in an actual situation (FC IJkdijk, 2016). The reason for starting this pilot project was that the assessment was not in accordance with the experience of the dike managers, and a quick study showed that some important data and information was not used. Furthermore, in December 2013 a huge storm hit the Netherlands, resulting in one of the highest water levels ever measured. Based on the live information from this event, and other information gathered over time it became quite clear that the dike was much stronger than appeared from the assessment, and also much stronger than initially expected by the experts. This is an example of improving the efficiency of a reinforcement measure by reducing uncertainty through SHM. In this paper a method is presented which enables assessing (part of) the benefits of SHM, specifically through measurements of the hydraulic head. First the principle of head monitoring is explained, after which a method is presented for assessing costs and benefits in a lifecycle approach. Based on the case study some conclusions are drawn on the use of Structural Health Monitoring in the lifecycle of a dike, and specifically on the case of the Ommelanderzeedijk.

### 2 MONITORING AND ITS RELATION TO DIKE STABILITY

#### 2.1 Geohydraulic failures

Next to overtopping, geohydraulic failures due to internal erosion and instability are the most recorded failures of dikes (Vorogushyn et al., 2009). Such failures are forced by high water pressures inside the dike, causing either material transport or loss of stability. Such high water pressures can be assessed by monitoring the hydraulic head in the dike. In general the behavior of the hydraulic head, given a certain load combination can vary greatly per dike and is highly uncertain due to a large influence of the structure of the subsoil.



Figure 1. Slope stability failure (Zwanenburg et al., 2013)

#### 2.2 Calculation of failure probability

The slope stability of a dike is typically expressed in terms of a stability factor (SF) that can be computed using e.g. limit equilibrium models. The SF highly depends on the level of the phreatic line in the dike body. For this study, we use a simplified approach for the relation between phreatic line and stability factor:

$$SF_{kd}(x) = C_1 * \ln(x) + C_2$$
 (1)

Where  $C_1$  and  $C_2$  are constants and x is the level of the phreatic line in m NAP, where NAP is the Dutch reference level.

Thus, for every scenario a factor of stability can be calculated. (Hoffmans, 2007) gives a standard relation between factor of stability and the reliability index:

$$\beta = A_1 + \frac{SF_{kd} - 1}{A_2} \tag{2}$$

Where  $A_1$  and  $A_2$  are constants,  $SF_{kd}$  is the characteristic factor of stability and  $\beta$  is the reliability index. For the constants it holds that  $A_1$ =4 and  $A_2$ =0.13. However, these are standard constants and these will in reality vary per location and should be derived from a local probabilistic analysis.

#### 2.3 Scenarios to address subsoil uncertainties

Due to the large uncertainty in subsoil build-up, in order to calculate failure probabilities for geohydraulic failure mechanisms a scenario approach can be used. In this approach, different subsoil scenarios are defined, each possibilities given the available measurements from for instance cone penetration tests. In a failure probability calculation such subsoil scenarios are weighed using scenario probabilities, using the following relation:

$$P_f = \sum_i P_{f,i} P(S_i) \tag{3}$$

Where  $P_f$  is the total failure probability, which is the sum of the products of scenario probability  $(P(S_i))$  and failure probability given scenario i  $(P_{f,i})$ for all scenarios. Due to obtained information by measuring and monitoring, some scenario probabilities might be reduced, resulting in a change in total failure probability.

As the behavior of the phreatic level in a dike is very dependent on the load condition and the structure of the subsoil inside the dike, monitoring this level can also lead to excluding or reducing the probability of certain scenarios. When applying head monitoring, the phreatic level can be related to the outside water level (if necessary combined with other forcings such as rainfall), which gives a relation between observed load and observed phreatic level.

As each subsoil scenario implicates a certain phreatic level (with some (measurement) uncertainty), monitoring can result in observations that contradict certain scenarios. By using Bayesian updating such observations can be implemented in the probability calculation, resulting in a more realistic failure probability estimate, based on actual observations.

# 2.4 Implementing observations in a scenario approach

The next step is to connect observations in the field to the different possible scenarios. This means that based on the observations scenarios can be more or less likely, and estimates of scenario probabilities can be based on evidence gathered by monitoring. In this study a simplified approach is taken towards the behavior of the phreatic level given a certain water level. Three linear relations with different responses are assumed, resulting in different phreatic



Figure 2. Different scenarios for the level of the phreatic line at daily (dotted) and extreme (solid) circumstances

levels at design conditions, but similar levels under daily circumstances. It is assumed that the build-up of the dike is sufficiently simple to be able to measure the phreatic line based on 1 measurement point. This results in the following formula for the response of the phreatic line given a certain outside water level:

$$\mathbf{x}_{\text{scen,fl}} = \mathbf{h}_{\text{fl,scen}} + \delta \mathbf{h}_{\text{fl}} + FR(\mathbf{h} - \mathbf{h}_{\text{fl,scen}})$$
(4)

Where  $x_{scen,fl}$  is the positioning of the phreatic line given water level h [m NAP],  $h_{fl,scen}$  is the base postion of the phreatic line [m NAP],  $\delta h_{fl}$  the rain infiltration [m] and *FR* the response given a high water [-].

Implementation of measurements in probability distributions or scenario probabilities can be done by Bayesian updating. Assuming an a priori distribution  $P(\theta)$  of the scenario probability at t=0, gathering observations  $x_n$  at t = n, will yield an a posteriori distribution  $P(\theta|x_n)$ , for which it holds that:

$$P(\theta \mid x_n) = P(\theta)f(x_n \mid \theta)$$
(5)

Where  $f(x_n|\theta)$  is the likelihood function. For t = n+1 it holds that  $P(x_{n+1}) = P_n(\theta)f(x_{n+1}|\theta)$ . So the posteriori distribution at t = n is the priori distribution for t = n+1. Such, monitoring data can be used to gradually update the scenario probabilities.

When monitoring hydraulic heads, in general measurements are taken under daily circumstances (Serviceability Limit State). In a scenario approach, this could mean that a measured water and phreatic level at daily circumstances already give a clear distinction between the scenarios. However, the scenarios are meant to representatively cover the range of possible phreatic levels, and are not three distinctive possible truths. Especially as dikes can behave very discontinuous (i.e. sudden sharp increases in phreatic level due to heterogeneity). Therefore there is additional extrapolation uncertainty when drawing conclusions regarding the Ultimate Limit State, given observations in the Serviceability Limit State. This is illustrated in Figure 2, where under daily circumstances (dotted line) the differences are minor, but

under extreme circumstances (solid line) the differences are significant.

Assuming a normal distribution for the likelihood function yields the following formula:

$$L_{scen}(x_{scen,fl},\sigma_{L} | x_{fl}) = (2\pi\sigma_{L})^{-\frac{1}{2}} * e^{-\frac{1}{2^{*}\sigma_{L}^{2}} * (x_{fl} - x_{scen,fl})^{2}}$$
(6)

Where,  $x_{fl}$  is the measured phreatic level,  $x_{scen,fl}$  is the phreatic level according to the scenario.  $\sigma_L$  is the standard deviation of the likelihood function. In order to account for the extrapolation uncertainty this standard deviation can be assumed dependent on the distance of the measured water level to the design water level, using the following formula:

$$\sigma_{\rm L} = \sigma_{\rm meas} + (h_{\rm max} - h_{\rm meas}) * f_{\rm extr}$$
(7)

Where  $\sigma_{meas}$  is the measurement uncertainty of head monitoring equipment (typically 20 cm), h<sub>max</sub> is the design water level, h<sub>meas</sub> is the measured water level and fextr is a factor for the extrapolation uncertainty. In this case fextr was assumed to be 0.5 based on expert judgement and analysis of the findings during the monitoring project. It has to be noted that this is a very case-specific relation, and it cannot be used for other cases. Figure 3 shows an example of the relation between priori (dashed line) and updated scenario probability (dots) for a single observation, in a case where the actual scenario is 3 and all priori scenario probabilities are 1/3. It can be seen that Scenario 2 is very distinctive and contradicts observations; therefore the probability is rapidly reduced. Scenarios 1 and 3 are more similar, and it requires a certain water level for  $P(S_3)$  to become equal to 1 (approximately 5 m NAP). Multiple measurements lower than 5 meters will also yield  $P(S_3)$ , as uncertainty is averaged.





Figure 3. Difference between priori(dashed lines) and posteriori (dots) scenario probabilities for single observations of different water levels for phreatic line scenario 3.

#### 3 MODEL & CASE STUDY

#### 3.1 Introduction

Using the previously outlined steps monitoring can be included in life-cycle analysis of a flood defence and its influence on costs and risks during the lifecycle, given a certain strategy, can be estimated, enabling evaluating long term benefits of different strategies. In a case study the influence of SHM on long term investments is considered. Three cases are considered: the first two cases consider a sea dike that failed the safety assessment ('rejected' or 'disapproved'), comparable to the Ommelanderzeedijk in the introduction. In the first case the costs and benefits of a monitoring action are assessed afterwards, so it is a hindcasting analysis, which provides insight in the benefits of the SHM at the Ommelanderzeedijk. The second case forecasts costs and benefits of the same monitoring action, so it gives insight in what would result from a comparison before starting the SHM-program. The third case considers a similar dike, but in a different life-cycle stage, namely right after reinforcement. In that case the dike is well above the minimum required performance level. For the scenarios of the phreatic level three scenarios are assumed, which are listed in Table 1.

Table 1. The three scenarios used for the phreatic line.

Scenario	h <sub>fl,scen</sub> [-]	δh <sub>fl</sub> [-]	FR [-]	phreatic level at design water level [m NAP]
1	1	0.5	0.6	4.5
2	1	0.5	0.8	5.5
3	1	0.25	0.25	2.5

#### 3.2 Strategies for life-cycle management

The main question of this case study is what would be a good strategy to implement structural health monitoring in the life-cycle of a dike, for instance: how long should we monitor, and at which point during the life-cycle? In the first two cases ,two strategies are considered: the first is the 'usual' approach, where the dike is reinforced after being disapproved, monitoring is not considered in this strategy. In the second strategy 'project monitoring' is considered: after disapproval, SHM is carried out in order to reduce uncertainty before reinforcement. Downside is that this delays the reinforcement by approximately 1 year, meaning that the dike is in a disapproved state for an extra year compared to the first strategy.

In the third case, due to the dike being in a different life-cycle stage, another strategy is possible, namely monitoring before disapproval. In this third strategy the dike is monitored for a longer period of time during the life-cycle, well before disapproval.



Figure 4. Influence of monitoring and reinforcement on performance.

The advantage is that this gives more time for monitoring, which increases the probability of an extreme water level occurring during the monitoring period, and hence the amount of information obtained will be higher. Due to the decreased time pressure the annual costs of monitoring will also be lower, in this case it is assumed that they will be  $1/3^{rd}$  of the annual costs for strategy 2. Strategies comprise of certain rules for decisions, resulting in the simulated performance over time, as shown in Figure 4. Here it can be seen that the information obtained by monitoring can result in postponement or acceleration of a reinforcement, due to a change in performance estimate.

#### 3.3 Comparison of strategies

The comparison of strategies can be done based on costs, risks and performance during the life-cycle. However, as this study considers investments over a longer period of time, these have to be made comparable, which can be done by discounting the costs in order to calculate their net present value, which is the value of the investment at the current day. This enables calculation of the total costs and benefits of an investment, over the life-cycle (Garvin and Cheah, 2004). The same approach can be used for the risk: as this is the virtual yearly cost of failures. Combining Net Present Cost and Risk to Net Present Value yields the following formula:

$$NPV = \sum_{n=1}^{N} \frac{\mathbf{I}}{(1+r)^{n}} + \sum_{n=1}^{N} \frac{P_{f}D}{(1+r)^{n}}$$
(8)

Where *NPV* is the Net Present Value in  $\in$ ; *I* is an investment (e.g. dike reinforcement) in  $\in$ ; *r* is the discount rate;  $P_f$  is the annual failure probability of the dike section and *D* is the damage of a failure in  $\in$ ; The NPV is calculated by summing up costs of all *N* years. For *N*, 200 years is used, as investments after this time horizon do not have any influence on the results. The principle of Net Present Value means that SHM can result in three types of benefits: a cheaper reinforcement due to reduced uncertainty, postponing a reinforcement due to higher estimated failure probability and reduction of risk due to earlier detection of lower than expected dike strength.

As there are different subsoil scenarios, in each scenario strategies will result in different risks and investments. These can be weighed by using a Bayesian pre-posterior analysis (Ben-Zvi et al., 1988; Klerk et al., 2015), in which the outcomes of the different scenarios are weighed based on their a priori probabilities. Thus the average expected Net Present Value can be calculated for a strategy, and compared with that of another strategy.

#### 3.4 Including uncertainty in measurements

One of the pivotal aspects of analyzing long term benefits of SHM for dikes, is the fact that the expected value of information is rather uncertain, as it is highly dependent on whether an extreme event is recorded or not. This is simulated by generating yearly values from an extreme value distribution of the water levels, which gives an annual maximum water level. Figure 5 shows an example generation of 10 time series of 10 years. If there would be monitoring during these 10 years, the black scenario would give a lot of information in year 5, the green would give a lot of information in year 8, but before the added value of monitoring is very limited due to a lack of observed extremes. By using a Monte Carlo realization of such time series the uncertainty in water levels is simulated, resulting in uncertain benefits and thus an uncertain Net Present Value.



Figure 5. Realizations for water levels as simulated in the model.

## 4 RESULTS

#### 4.1 *Case 1: Disapproved sea dike: hindcasting*

The first case is a hindcasting study of the actual benefits at a simplified version of the Ommelanderzeedijk. In this case two strategies are compared as mentioned in section 3.2. Table 2 shows the a priori scenario probabilities that are used.

Table 2. Scenario probabilities for case 1 for both strategies

Scenario	A priori scenario probability		
	Strategy 1	Strategy 2	
1	0.05	0.6	
2	0.9	0.2	
3	0.05	0.2	

In this case different scenario probabilities are assumed for the scenarios. This can be done, as a hindcasting study is done with a fixed scenario. In this case the 'actual scenario' was scenario 3. However, if strategy 1 would have been fully executed this would imply that the assessment (scenario 2) would be believed, which was not the case. To include the benefits of 're-evaluating' the assessment result, for strategy 2 different scenario probabilities are used.

The Net Present Value over a period of 200 years is between 5 and 35% lower for strategy 2 than for

strategy 1, meaning that the additional information obtained by monitoring gives a significant reduction in overall cost and risk. Figure 6 shows the direct costs of the first reinforcement for both strategies.

From this, in the wide range of possible reinforcement costs for strategy 2 (between 5 and 12  $M \in$ ), the influence of uncertainty in value of information from monitoring can be clearly observed. Due to this uncertainty in some cases monitoring might not lead to any benefits (e.g. if no high water level is recorded), or it might reduce reinforcement costs by around 50%. As in the actual case, the

measured water level was extremely high, it is estimated that the benefits of monitoring at the Ommelanderzeedijk, in this context, would be at least around 40% of the reinforcement costs. So for this particular case monitoring has clear benefits.



Figure 6. Direct cost of the first reinforcement for strategy 1 (grey), and strategy 2 (white).

#### 4.2 Forecasting

In order to forecast the benefits of different strategies a Bayesian pre-posterior analysis is used. For a preposterior analysis, in order to make a valid comparison between strategies, the same priori probabilities have to be used, as the results are weighed based on these probabilities. In this case, the scenario probabilities for scenarios 1 to 3 are 0.6, 0.2 and 0.2 respectively. Figure 7 shows the Net Present Values for 200 years for both strategies for each scenario. For Scenario 3, it can be seen that the shaded area is slightly left of the dashed line, meaning that even though the reinforcement would need to be postponed, it would still yield benefits. For Scenario 1 this is not the case, and for Scenario 2 it is clear that postponing the reinforcement in order to monitor is not cost-efficient, as the additional risk is much higher than the expected benefits (i.e. shaded area is right of the dashed line).

If the three scenarios are summed up based on their scenario probabilities the 200-year NPV for strategy 1 would be 15.2(10.8/25.9) M $\in$ , while the NPV for strategy 2 would be 20.6(9.9/40.7) M $\in$ , where the values between brackets denote 5/95%-values.

From the results from this pre-posterior analysis it is shown that the benefits beforehand were not that clear, which raises the question whether the analysis sufficiently covers all benefits, and if the situation at the time it was decided to start a monitoring program was correctly represented in the model. However, from the analysis it can be seen that in cases where the risk is still relatively low, project monitoring can yield considerable benefits, but that for some cases taking extra risk to monitor is not cost-efficient. Also it has to be noted that only a part of the benefits of monitoring is considered.



Figure 7. Total Net Present Value for the two strategies for three scenarios. Dashed lines denote the NPV for strategy 1, shaded areas denote the (uncertain) NPV for strategy 2.

#### 4.3 *Lifecycle monitoring*

Based on the findings of the forecasting example a third case was studied, again with the same dike and same scenarios, but now in a different stage of the life-cycle. This enables a new strategy: lifecycle monitoring, where a monitoring project is not carried out immediately before reinforcement, but during the lifecycle in order to prevent time pressure and postponement of the reinforcement. The initial situation at t=0 is a dike which has been reinforced approximately 25 years ago and will be reinforced in approximately 25 years. Figure 10 shows the performance in time for this strategy. The black line representing the strategy without monitoring shows a regular interval for reinforcement, whereas the for instance the red dotted line shows a clear 'learning effect' between year 10 and 16, where the estimated factor of stability is increased from 1.06 to 1.16, resulting in a postponement of the reinforcement by 47



Figure 10 Performance in time for two strategies: Lifecycle monitoring and normal reinforcement. By the gradual changes in the red lines the influence of monitoring can be clearly observed.



Figure 9 Total Net Present Value for lifecycle monitoring (shaded areas), and normal reinforcement (dotted lines).

years. Figure 10 shows the Net Present Value per scenario. It can be seen that for scenario 1, there is no benefit by monitoring. This is due to the fact that the initial averaged failure probability, based on weighing the scenarios is approximately equal to the failure probability in scenario 1. The benefits of reducing scenario uncertainty in general are not taken into account in the model. For scenario 2 it is signaled that the dike is weaker than expected, but as the risk is much lower than the cost for monitoring and the extra cost due to the fact that the dike is reinforced earlier, the total Net Present Value is much higher. In scenario 3, due to a large postponement of the reinforcement (also observed in Figure 10), the Net Present Cost are reduced significantly, resulting in a lower Net Present Value.



Figure 9. Total Net Present Value for lifecycle monitoring (shaded areas), and normal reinforcement (dotted lines) with increased risk.

#### 4.4 Influence of balance in cost and risk

From the results in the preceding paragraph it becomes clear that a pivotal factor in the preposterior analysis is the balance between costs and risk. In the cases, the risk was very small, relative to the cost, meaning that the influence of the cost, dominated the analysis. For instance, in Figure 9, the risk is reduced by approximately 40% for scenario 2, but the cost at the same time increases by approximately 40%. In order to put this in perspective a sensitivity analysis was done where the damage was multiplied by a factor 10, in order to achieve a better balance between risk and cost. It should be noted that the safety standard should be based on this balance in order to ensure optimal investments. Figure 9 shows an adapted version of the third case, where the damage is multiplied by 10. The principle of the figure is the same as for Figure 9, but it can be seen that especially for scenario 2, due to the changed balance in cost and risk, the strategies now have approximately the same expected Net Present Value.

# 5 DISCUSSION & CONCLUSIONS

From the analysis it was shown that the monitoring experiment studied resulted in a cost saving of approximately 40%. However, from the preposterior analysis the benefits are less clear, as in some scenarios the benefits are minor or are smaller than the additional risk. There are several reasons for this, but the most important is that not all benefits are taken into account. In the model only the effect on reinforcement costs is taken into account. Also, while monitoring greatly improves insight in the dike behavior, also in extreme situations, this is not necessarily reflected in more efficient long term investments, as the performance might still be the same. Whether monitoring is a useful addition is determined by the balance between costs and risks, the time available and the amount of uncertainty associated with the scenarios. Also, while in this model the reinforcement costs are determined by the average failure probability, usually a high uncertainty in subsoil scenarios will also result in a much more conservative design. The reduction in uncertainty is not fully taken into account in the model.

The model builds upon the assumption that it is possible to set up a good monitoring campaign, which always requires insight in the buildup of the subsoil. Due to this, setting up a monitoring campaign already brings order in available data and structures the process of uncertainty reduction, not just by monitoring itself. In the model the long term benefits of various monitoring projects are estimated, but not all benefits have been included (yet). Other benefits, such as live insight in emergency situations and less uncertainty in general will also result in a better general performance and lower costs and risks.

In general it can be concluded that monitoring will have clear benefits in long term investments, but that not all benefits are quantified in the model. Therefore the model should be used as an aid in quantifying expert feeling before a decision on whether to monitor or not. It should be improved on better relating cost savings in design to uncertainty reduction, which can be done by using a probabilistic failure model. The adapted likelihood function for taking extrapolation uncertainty into account works quite well for this case study, but should be made case and scenario-specific.

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